N-CYLINDRICAL FUZZY BIPOLAR INTERVAL NEUTROSOPHIC TOPLOGICAL SPACE

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Abstract

A fuzzy set theory extend classical set theory by allowing elements graded membership level between 0 and 1, formalized through membership functions. Evolutionary fuzzy concept have been increased with the model of interval fuzzy, neutrosophic, interval neutrosophic, neutrosophic topological space etc. Our work is an attempt to define n-Cylindrical fuzzy bipolar interval neutrosophic sets and topology as the largest extension of fuzzy sets and fuzzy topology to deal with the problems involved periodicity information and varies with time in, bipolar interval forms. In this paper, a representation n-cylindrical fuzzy bipolar interval neutrosophic based on the real numbers in the bipolar interval form is proposed. Then, we provide some of its basic properties, namely union, intersection and complement. Also n-cylindrical fuzzy bipolar interval neutrosophic topological space is introduced. By defining n-cylindrical fuzzy bipolar interval neutrosophic topology, we give some results in the form of theorems and propositions. Further, some examples are given to justify the definitions defined in this work.

Keywords: Cylindrical fuzzy sets, topology space, neutrosophic sets, interval neutrosophic sets, bipolar neutrosophic set.

الفضاء التوبولوجي لدالة النونية التكرارية الضبابية لنتروفزك ذو الفترة ثنائية القطب

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المستخلص:

الدالة الضبابية امتداد للدالة الاساسية والتي عناصرها بين 0 و1. ولقد تطورالنضام الضبابي بعد تعريف الدالة الضبابية ذات الفترة ودالة نتروفزك ذات الفترة.وتبولوجيا دالة النتروفزك. الهدف من هذا العمل هوتعريف ودراسة الدالةنتروفزك الضبابية ذات التكرار نون والفترة وكذلك التوبولوجيا على الدالة المذكورة اعلاه كأكبر اشتقاق لدالة الضبابية وتوبولوجيا الدالة الضبابية لتعامل مع المعلومات ذات القطبين والفترة.حيث قمنا بتعريف العمليات الاساسية لدالة كوكذلك دراسة وتعريف التوبولوجيا عليها مع دراسة العديد من النظريات والخصائص المتعلقة بها مع التوضيح بالامثلة.

الكلمات المفتاحية: الدالة الضبابية التكرارية،توبولوجي الدالة الضبابية ، دالة نتروفزك، دالة نتروفزك ذات القطبين ، الفضاء التوبولوجي الضبابي.

1.Introduction

In 1965 L.A. Zadeh defined fuzzy sets. The idea of fuzzy set emerged as a novel tool to deal with uncertainties in real-life problems, and he just discussed the membership function. (Atanassov ,1986) extend traditional fuzzy sets by introducing a new set called intuitionistic fuzzy set, which its non-membership grade plus its membership grade, does not exceed 1. Many authors have been studied these sets models; on theory and applications and so on (Fatma & Hassan 2014, 2015 2016). On other hand, these sets are failed to deal with all date of uncertainty, such as inconsistent and indeterminate in many decision-making situations. To overcome of this problem, (Smarandache 1999) established new model, which makes (Atanassov1986) very practical and useful theory to apply. During current decade , the environments of neutrosophic are mainly interested by researchers in different fields. (Wang et al. 2005).] defined interval neutrosophic sets, while (Delia et al. 2015) introduced the idea of bipolar neutrosophic sets, as an generalization of neutrosophic sets. In hybrid of bipolar and neutrosophic model, the positive membership degree

$T+(x)$,$I+(x)$,$F+(x)$ and the negative membership degree $T-(x)$,$I-(x)$,$F-(x)$ of an element x ∈ X to some implied counter-property analogous to a bipolar neutrosophic set. Other investigations followed defining neutrosophic set and interval neutrosophic set such as (Kumari et al. 2022, Lupianez 2009, Maji 2013, Smarandache 2005, Ye 2014, Zhang et al,.2014).The concept of fuzzy topology was established by(Chang 1967). On other hand, the idea of NTS proposed by (Salama & Alblowi 2012), where the properties of generalised closed sets were discussed on NTS. Also, the relation between topology and interval value studied by (Lupianez 2009) . Recently, several researches developed the generalizations of the concept of fuzzy sets and NTS as (Arokiarani et al. 2013,2014, Nanthini & Pushpalatha 2020, Veereswari 2017). From the literature survey, it was noticed that precisely the concept of n- Cylindrical fuzzy bipolar interval neutrosophic set and topology are not performed. Then, as an update for the research in topology and interval value neutrosophic sets,we define the concept of n- Cylindrical fuzzy bipolar interval neutrosophic set and discuss its basic properties. Also, we form a topological structure on n- Cylindrical fuzzy bipolar interval neutrosophic and establish its properties with supporting proofs and illustrating examples.

2.Preliminaries

Some definitions and basic operations related to fuzzy set and neutrosophic set are recalled in this section.

Definition 2.1 (Zadeh 1956). A fuzzy set A in U is defined by membership function $\mu: X \to [0,1]$whose membership value $\mu_A(x)$ shows the degree to which $x \in U$ includes in the fuzzy set A for all $x \in U$.

Definition 2.2 (Smarandache 1999). A neutrosophic set A on U is $A = \{T_A(x), I_A(x), F_A(x) >; x \in U\}$ where $T_A(x), I_A(x), F_A(x): A \to]-0, 1^+[$ and -0 < $T_A(x), I_A(x), F_A(x)$< $n3^+$.

Definition 2.3 (Wang et al., 2005)**.** An interval value neutrosophic set (IVN-sets) A in U is characterized by truth-membership function T_A, an indeterminacy-membership function I_A and a falsity-membership function F_A. For each point $x \in U$; $T_A(x), I_A(x), F_A(x) \subseteq [0,1]$.

Definition 2.4 (Fatma et al, 2023). An n- Cylindrical fuzzy interval neutrosophic A on U is an object of the form $A =< x, [\alpha_A(x)^L, \alpha_A(x)^U], [\beta_A(x)^L, \beta_A(x)^U], [\gamma_A(x)^L, \gamma_A(x)^U] : x \in U >$ where $\alpha_A{}^{LU}(x) \in [0,1]$ called the degree of positive membership of x in A,

$\beta_A{}^{LU}(x) \in [0,1]$, called the degree of neutral membership of x in A and $\gamma_A{}^{LU}(x) \in [0,1]$ called the degree of negative membership of x in A, which satisfies the condition, (for all $x \in U$) , $0 \leq \beta_A(x)^L, \beta_A(x)^U \leq 1$ and $0 \leq \alpha_A n(x)^L + \gamma_A n(x)^L \leq 1$, $0 \leq \alpha_A n(x)^U + \gamma_A n(x)^U \leq 3, n > 1$, is an integer. Here T and F are dependent interval neutrosophic components and I is 100% independent. For the convenience, $[\alpha_A(x)^L, \alpha_A(x)^U], [\beta_A(x)^L, \beta_A(x)^U], [\gamma_A(x)^L, \gamma_A(x)^U]$ is called as nCylindrical fuzzy interval neutrosophic number (n-CyFINN) and is denoted as

$$A = <\alpha_A{}^{LU}, \beta_A{}^{LU}, \gamma_A{}^{LU}>.$$

Definition 2.5 (Nanthini & Pushpalatha 2020). An interval valued neutrosophic topological space of interval valued neutrosophic set (In short IVN topological space) is a pair$(X, t_N))$ where X is a nonempty set and t_N is a family of IVN sets on X satisfying the following axioms:

$1 - 0_N, 1_N \in t_N$.

$2 - A, B \in t_N \Rightarrow A \cap B \in t_N$.

$3 - A_i \in t_N, i \in I \Rightarrow \cup_{i \in I} A_i \in t_N$.

3. N- Cylindrical Fuzzy Bipolar Interval Neutrosophic Set.

With some related results and illustrating examples, we propose the notion of n- Cylindrical fuzzy bipolar interval neutrosophic set.

Definition 3.1. The n- Cylindrical fuzzy bipolar interval neutrosophic set Aon U defined as

$$A = < x, [\alpha_A(x)^L, \alpha_A(x)^U]^+, [\beta_A(x)^L, \beta_A(x)^U]^+, [\gamma_A(x)^L, \gamma_A(x)^U]^+ ,$$
$$[\alpha_A(x)^L, .\alpha_A(x)^U]^-, [\beta_A(x)^L, \beta_A(x)^U]^-, [\gamma_A(x)^L, \gamma_A(x)^U]^- : x \in U >,$$

where $\alpha_A{}^{LU+}(x) \in [0,1]$and $\alpha_A{}^{LU-}(x) \in [-1,0]$ called the degree of positive membership of x in A, $\beta_A{}^{LU+}(x) \in [0,1]$ and $\beta_A{}^{LU-}(x) \in [-1,0]$ called the degree of neutral membership of x in A and $\gamma_A{}^{LU+}(x) \in [0,1]$ and $\gamma_A{}^{LU-}(x) \in [-1,0]$ called the degree of negative membership of x in A, such that, (for all $x \in U$) , $0 \leq \beta_A(x)^L, \beta_A(x)^U \leq 1$ and $0 \leq \alpha_A n(x)^L + \gamma_A n(x)^L \leq 1$, $0 \leq \alpha_A n(x)^U + \gamma_A n(x)^U \leq 3, n > 1$, $-1 \leq \beta_A(x)^{L-}, \beta_A(x)^{U-} \leq 0$ and $-1 \leq \alpha_A n(x)^{L-} + \gamma_A n(x)^{L-} \leq 0$, $-3 \leq \alpha_A n(x)^U + \gamma_A n(x)^U \leq 0, n > 1$, is an integer number. Also Truth and False are dependent interval neutrosophic components and I is 100% independent. For the convenience,

$[\alpha_A(x)^L, \alpha_A(x)^U]^+, [\beta_A(x)^L, \beta_A(x)^U]^+, [\gamma_A(x)^L, \gamma_A(x)^U]^+$, $[\alpha_A(x)^L, \alpha_A(x)^U]^-, [\beta_A(x)^L, \beta_A(x)^U]^-, [\gamma_A(x)^L, \gamma_A(x)^U]^-$

is the n-Cylindrical fuzzy bipolar interval neutrosophic number (n-CyFBINN) and define as

$A =< {\alpha_A}^{LU}, {\beta_A}^{LU}, {\gamma_A}^{LU}, {\alpha_A}^{LU-}, {\beta_A}^{LU-}, {\gamma_A}^{LU-} >.$

Example 3.1 Let, $U = \{u_1, u_2, u_3\}$. The shape is u_1, the weight is u_2 and the color is u_3. The u_1, u_2 and u_3 values are between 0 and 1.Under domain experts ' questionnaire, the choices of their opinions are goodness degree, indeterminacy degree and poorness degree. A is an n-CyFBNISs of U given by

$A =$
$< u_{1,}([0.2,0.4], [0.3,0.5], [0.3,0.5], [-0.1,0], [-0.3, -0.3], [0, -0.4]), u_2,$
$([0.5,0.7], [0,0.2], [0.2,0.3], [-0.3, -0.2], [-0.4, -0.2], [-0.5, -0.5])u_3, ([0.6,0.8],$
$[0.2,0.3], [0.2,0.3], [-0.5,0], [-0.4, -0.2], [-0.1,0]) >$.And

$B =$
$< u_{1,}([0.1,0.3], [0.3,0.5], [0.2,0.5], [-0.3,0], [-0.3, -0.3], [-0.1, -0.2]),$
$u_2, ([0.5,0.5], [0,0.1], [0.3,0.3], [-0.3, -0.3], [-0.4, -0.1], [-0.4, -0.2])$
$u_3, ([0.1,0.8], [0.1,0.3], [0.2,0.3], [-0.4, -0.3], [-0.5, -0.4], [-0.1,0]).$

Definition 3.2. For every n-CyFBNISs the complement of an n-CyFBNISs A

is $\alpha_A(x)^L = 1 - \alpha_A(x)^L$, $\alpha_A(x)^U = 1 - \alpha_A(x)^U$, $\beta_A(x)^L = 1 - \beta_A(x)^L$, $\beta_A(x)^U = 1 - \beta_A(x)^U$, $\gamma_A(x)^L = 1 - \gamma_A(x)^L$, $\gamma_A(x)^U = 1 - \gamma_A(x)^U$, $\alpha_A(x)^{L-} = 1^- - \alpha_A(x)^{L-}$, $\alpha_A(x)^{U-} = 1^- - \alpha_A(x)^{U-}$, $\beta_A(x)^{L-} = 1^- - \beta_A(x)^{L-}$, $\beta_A(x)^{U-} = 1^- - \beta_A(x)^{U-}$, $\gamma_A(x)^{L-} = 1^- - \gamma_A(x)^{L-}$ and $\gamma_A(x)^{U-} = 1^- - \gamma_A(x)^{U-}$.

Definition 3.3. If A and B are two n-CyFBNISs. Then the union of them is

$A \cup B =$

$$\{u, [\max(\alpha_A(u)^L, \alpha_B(u)^L), \max((\alpha_A(u)^U, \alpha_B(u)^U)]^+, [\frac{\beta_A(u)^L + \beta_B(u)^L}{2},$$

$$\frac{(\beta_A(u)^L + \beta_B(u)^L}{2}]^+, [\min(\gamma_A(x)^L, \gamma_B(x)^L), \min(\gamma_A(x)^U, \gamma_B(x)^U)]^+,$$

$[\min(\alpha_A(u)^L, \alpha_B(u)^L), \min((\alpha_A(u)^U, \alpha_B(u)^U)]^-, [\frac{\beta_A(u)^L + \beta_B(u)^L}{2},$
$\frac{(\beta_{A_1}(u)^L + \beta_B(u)^L}{2}]^-, [\max(\gamma_A(x)^L, \gamma_B(x)^L), \max(\gamma_A(x)^U, \gamma_B(x)^U)]^-$
$: u \in U\}.$

Example 3.2. Using **Example 3.1**. Then

$A \cup B =$
$\{u_{1,}([0.2,0.4], [0.3,0.5], [0.2,0.5], [-0.3,0], [-0.3, -0.3], [0, -0.2]$
$u_2, ([0.5,0.7], [0,0.15], [0.2,0.3], [-0.3, -0.3], [-0.4. -0.15], [-0.4, -0.2]),$
$u_3, ([0.6,0.8], [0.15,0.3], [0.2,0.3], [-0.4,0], [-0.45. -0.3], [-0.1,0]),$

Definition 3.4. . If A and B are two n-CyFBNISs. Then the intersection of them is

$A \cap B = \{u, [\min(\alpha_{A_1}(u)^L, \alpha_B(u)^L), \min((\alpha_{A_1}(u)^U, \alpha_B(u)^U)]^+,$
$[\frac{\beta_{A_1}(u)^L + \beta_B(u)^L}{2}, \frac{(\beta_{A_1}(u)^L + \beta_B(u)^L}{2}]^+, [\max(\gamma_A(x)^L, \gamma_B(x)^L), \max(\gamma_A(x)^U,$
$\gamma_B(x)^U)]^+, [\max(\alpha_{A_1}(u)^L, \alpha_B(u)^L), \max((\alpha_{A_1}(u)^U, \alpha_B(u)^U)]^-, [\frac{\beta_{A_1}(u)^L + \beta_B(u)^L}{2},$
$\frac{(\beta_{A_1}(u)^L + \beta_B(u)^L}{2}]^-, [\min(\gamma_A(x)^L, \gamma_B(x)^L), \min(\gamma_A(x)^U, \gamma_B(x)^U)]^-: u$
$\in U\}.$

Example 3.3. Using **Example 3.1**. Then

$A \cap B = \{u_{1,}, ([0.1,0.3], [0.3,0.5], [0.3,0.5], [-0.1,0], [-0.3, -0.3]),$
$[-0.1, -0.4]), u_2, [0.5,0.5], [0,0.15], [0.3,0.3], [-0.2, -0.3],$ [-0.4,-0.15],
$[-0.5, -0.5]), u_3$,([0.1,0.8],[0.15,0.3],[0.2,0.3],[-0.4,-0.2],[-0.45,-0.3],
[-0.1,0])}.

Definition 3.5. Let A and B are two n- CyFBINS. Then

1- $A \subseteq B$ if

$\alpha_A(u)^{L+} \leq \alpha_B(u)^{L+}, \alpha_A(u)^{U+} \leq \alpha_B(u)^{U+}$ and $\beta_A(u)^{L+} \leq \beta_B(u)^{L+}$
$\beta_A(u)^{U+} \leq \beta_B(u)^{U+}$ and $\gamma_A(u)^{L+} \geq \gamma_B(u)^{L+}, \gamma_A(u)^{U+} \geq \gamma_B(u)^{U+}$. $\alpha_A(u)^{L-} \geq \alpha_B(u)^{L-}, \alpha_A(u)^{U-} \geq \alpha_B(u)^{U-}$ and $\beta_A(u)^{L-} \geq \beta_B(u)^{L-}$
$\beta_A(u)^{U-} \geq \beta_B(u)^{U-}$ and $\gamma_A(u)^{L-} \leq \gamma_B(u)^{L-}, \gamma_A(u)^{U-} \leq \gamma_B(u)^{U-}$.

for all u ∈ U.
2- A = B ↔ A ⊆ B and B ⊆ A.

4. N-Cylindrical Fuzzy Bipolar Interval Neutrosophic Topological Spaces

We introduce the notion of` n- CyFBINTS with some results and examples`.

Definition 4.1 The n -cylindrical fuzzy bipolar interval neutrosophic topological spaces ofn -CyFBINS (n-CyFBINTS) is a pair (U, t_{FBIN}) where U is a nonempty set and t_{FBIN} is a family of n-CyFBIN sets on U satisfying the following axioms:

1) $0_{FBIN}, 1_{FBIN} \in t_{FBIN}$.

2) $A, B \in t_{FBIN} \Rightarrow A \cap B \in t_{FBIN}$.

3) $A_\tau \in t_{FBIN}, \tau \in I \Rightarrow \cup_{\tau \in I} A_\tau \in t_{FBIN}$.

t_{FBIN} is called n-CyFBINTS on U and t_{FBIN} members are n-CyFBIN open sets.

Example 4.1. Let $U = \{u_1, u_2\}$ and

$A = < u_{1,}([0.2,0.4], [0.3,0.5], [0.2,0.5], [-0.3,0], [-0.3,-0.3], [0,-0.1]), u_2, ([0.5,0.7], [0,0.2], [0.2,0.3], [-0.3,-0.3], [-0.4,-0.2], [-0.4,-0.2]) >$,

$B = < u_{1,}([0.1,0.3], [0.3,0.5], [0.3,0.5], [-0.1,0], [-0.3,-0.3], [-0.1,-0.1]), u_2, ([0.5,0.5], [0,0.2], [0.3,0.3], [-0.3,-0.2], [-0.4,-0.2], [-0.5,-0.5]) >$,

$0_{FBIN} = \{[0,0], [1,1], [1,1], [0,0], [-1,-1], [-1,-1]\}$,

$1_{FBIN} = \{[1,1], [0,0], [0,0], [-1,-1], [0,0], [0,0]\}$, $t_{FBIN} = \{0_{FBIN}, 1_{FBIN}, A, B\}$ is a n-CyFBINTS on U.

Theorem 4.1. Let $\{t_{FBIN\tau}\ \tau \in I\}$ be a family of n-CyFBINTS of n-CyFBINS on U. Then $\cap_\tau \{t_{FBIN\tau}\ \tau \in I\}$ is also an n-CyFBINTof n-CyFBINS on U.

Proof. 1) $0_{FBIN}, 1_{FBIN} \in t_{FBIN\tau}$ for each $\tau \in I$, Hence$0_{FBIN}, 1_{FBIN} \in \cap_i \{t_{FBIN\tau}\tau \in I\}$.

2) Suppose $\{A_\tau, \tau \in I\}$ be arbitrary family of n-CyFBINS where $A_\tau \in \cap_\tau t_{FBIN\tau}$ for each $\tau \in I$.

3) For all $\tau \in I$, $A_\tau \in t_{FBNi}$, since $t_{FBIN\tau}$ is a CyFBINTS, therefore $\cup A_\tau \in t_{FBIN\tau}$ for all $\tau \in I$.Thus $\cup A_\tau \in \cap_\tau t_{FBIN\tau}$.

Definition 4.2 Let (U, t_{FBIN}) be an n-CyFBINTS. An n-CyFBIN set A of U is called an n-CyFBIN-closed set if the complement of A (A^c) is a n-CyFBIN open set in t_{FBIN}.

Example 4.2 From Example **4.1** we have, n-CyFBIN set
$C =< u_1, ([0.8,0.6], [0.7,0.5], [0.8,0.5], [-0.7, -1], [-0.7, -0.7],$
$(u_2, ([0.5,0.2], [1,0.8], [0.8,0.7], [-0.7, -0.7], [-0.6, -0.8], [-0.6, -0.8]) >$
is closed set in (U, t_{FBIN}).

Theorem 4.2 If (U, t_{FBIN}) be an n-CyFBINTS. Then

i) $0_{FBIN}, 1_{FBIN}$ are n-CyFBIN – closed sets.

ii) $\{\cap A_\tau : A_\tau \in t_{FBIN}\tau \in I\}$is closed set, where A_τ is n-CyFBIN – closed sets.

iii)$\{\cup A_\tau : A_\tau \in t_{FBIN}, \tau = 1,2, \dots, r\}$ is closed set of A_τ n-CyFBIN – closed sets

Proof. i) From **Definition4.2**, we have $0^c{}_{FBIN} = 1_{FBIN} \in t_{FBIN}$ and $1^c{}_{FBIN} = 0_{FBIN} \in t_{FBIN}$, hence $0_{FBIN}, 1_{FBIN}$ are n-CyFBIN – closed sets.

ii) Assume $\{A_\tau : \tau \in I\}$ be an arbitrary family of n-CyFBIN – closed sets, therefore $\{A_\tau{}^c : \tau \in I\}$ an arbitrary family of n-CyFBIN – open sets, and also $\cup A_\tau{}^c$ n-CyFBIN – open sets, but $\cup A_\tau{}^c = (\cap A_\tau)^c$. Thus$\{\cap A_\tau\}$is closed set.

iii) Conceder $\{A_\tau : \tau = 1,2, \dots, r\}$ is n-CyFBIN – closed sets. Then $\{A_\tau{}^c : \tau = 1,2, \dots, r\}$ is n-CyFBIN – open sets and $\cap A_\tau{}^c$ is n-CyFBIN – open sets, but $\cap A_\tau{}^c = (\cup A_\tau)^c$, therefore $\cup A_\tau$ is closed set.

Definition 4.3 Let (U, t_{FBN}) be an n-CyFBIN topological space. And A is n-CyFBIN set of U , the interior and closure of A is denoted by n-CyFBIN $int(A)$ and n-CyFBIN $cl(A)$ are defined as

n-CyFBIN $int(A) = \cup \{Q \in t_{FBN}: Q \subseteq A\}$

n-CyFBIN $cl(A) = \cap \{Q \in t^c{}_{FBN}: A \subseteq Q\}$.

Example 4.3 Let $U = \{u_1, u_2\}$ and $t_{FBIN} = \{0_{FBIN}, 1_{FBIN}, G_1, G_2\}$ be n-CyFBINTS where
$G_1 =$
$< u_{1,}([0.2,0.4], [0.3,0.5], [0.2,0.5], [-0.3,0], [-0.4, -0.2], [0, -0.1]), u_2,$

$([0.5,0.7],[0,0.2],[0.2,0.3],[-0.3,-0.3],[-0.3,0],[-0.4,-0.2])>$,
$G_2 =< u_{1,}([0.1,0.3],[0.3,0.5],[0.3,0.5],[-0.1,0],[-0.4,-0.2],$
$[-0.1,-0.1]), u_2([0.5,0.5],[0,0.2],[0.3,0.3],[-0.3,-0.2],[-0.3,0],$
$[-0.5,-0.5])>$, if

$\mathrm{A} =< \mathrm{u}_1, ([0.3,0.6],[0.1,0.5],[0.1,0.3],[-0.7,-1],[-0.7,-0.7],$
$[-1,-0.9]), u_2, ([0.3,0.2],[1,0.8],[0.8,0.7],[-0.7,-0.7],[-0.6,-0.8],$
$[-0.6,-0.8])>$, then n-CyFBIN $int(A) = G_1$
$=< u_{1,}([0.2,0.4],[0.3,0.5],[0.2,0.5],[-0.3,0],[-0.4,-0.2],[0,-0.1]), \mathrm{u}_2,$
$([0.5,0.7],[0,0.2],[0.2,0.3],[-0.3,-0.3],[-0.3,0],[-0.4,-0.2])>$,

and n-CyFBIN $cl(A)$=${G_1}^c$
$=< u_{1,}([0.8,0.6],[0.7,0.5],[0.8,0.5],[-0.7,-1],[-0.6,-0.7],[-1,-0.9]),$
$u_{2,}([0.5,0.2],[1,0.8],[0.8,0.7],[-0.7,-0.7],[-0.7,-1],[-0.6,-0.8])>$.

Theorem4.3 Let $(\mathrm{U}, \mathrm{t}_{\mathrm{FBN}})$ be an n-CyFBINTS. And C is n-CyFBINS of U then the following properties holds:

1) n-CyFBIN $\mathrm{int}(1-\mathrm{C}) = 1-(\mathrm{n}-\mathrm{CyFBINcl}(\mathrm{C}))$.
2) n-CyFBIN $\mathrm{cl}(1-\mathrm{C}) = 1-(\mathrm{n}-\mathrm{CyFBINint}(\mathrm{C}))$.

Proof. 1) By Definition 4.3 n-CyFBIN $int(C) =\cup \{Q \in t_{FBIN}: Q \subseteq C\}$,
$1-(\mathrm{n}-\mathrm{CyFBIN}int(C)) = 1 -\cup \{Q \in t_{FBIN}$ (open set): $Q \subseteq C\}$,
$=\cap \{1-Q \in {t^c}_{FBIN}$ (closed): $1-C \subseteq Q\}$,
$=\cap \{K \in {t^c}_{FBIN} : 1-C \subseteq K\}$,
$= \mathrm{n}-\mathrm{CyFBIN}cl(1-C)$.

2) From Definition 4.3 we have n-CyFBIN $cl(C) =\cap \{Q \in {t^c}_{FBIN}: C \subseteq Q\}$,
$1-(\mathrm{n}-\mathrm{CyFBIN}cl(C)) = 1 -\cap \{Q \in {t^c}_{FBIN}$ (closed set): $C \subseteq Q\}$,
$=\cup \{1-Q \in t_{FBIN}$ (open set): $Q \subseteq 1-C\}$
$=\cup \{K \in \mathrm{t}_{\mathrm{FBIN}}$ (open set) : $K \subseteq 1-C\}$,
$= \mathrm{n}-\mathrm{CyFBIN}int(1-C)$.

Theorem4.4 Let $(\mathrm{U}, \mathrm{t}_{\mathrm{FBN}})$ be an n-CyFBIN TS. And A and B are n-CyFBIN set of U then the following properties holds**:**

1) n-CyFBIN $\mathrm{cl}(0_{FBIN}) = 0_{FBIN}$.
2) n-CyFBIN $\mathrm{in}(1_{FBIN}) = 1_{FBIN}$.
3) n-CyFBIN $\mathrm{int}(\mathrm{A} \cap \mathrm{B})$ = n-CyFBIN $\mathrm{int}(\mathrm{A}) \cap$ n-CyFBINint(B).

4) n-CyFBIN cl(A ∪ B) = n-CyFBIN cl(A) ∪n-CyFBINcl(B).

Proof. 1) and 2) prove in similar way, From Definition 4.3, we get $0_{FBN} \subseteq$ n-CyFBINcl(0_{FBIN}) and n-CyFBINcl(0_{FBIN}) $\subseteq 0_{FBIN}$, thus n-CyFBIN cl(0_{FBIN}) = 0_{FBIN}.

3) n-CyFBIN int(A ∩ B) implies that n-CyFBIN int(A ∩ B) ⊆ n-CyFBIN int(A) and n-CyFBIN int(A ∩ B) ⊆ n-CyFBIN int(B) , we obtain n-CyFBIN int(A ∩ B) ⊆ n-CyFBIN int(A) ∩ n-CyFBIN int(B).On the other hand, n-CyFBIN int(A) ⊆ A, and n-CyFBIN int(B) ⊆ B, then n-CyFBIN int(A) ∩ n-CyFBIN int(B) ⊆ $(A \cap B)$,and n-CyFBIN int(A) ∩ n-CyFBIN int(B) ⊆ $n -$ CyFBIN int(A ∩ B), for which we obtain the required result.

4) Since n-CyFBIN(A) ⊆ n-CyFBINcl(A) and n-CyFBIN(B) ⊆ n-CyFBIN cl(B). Thus n-CyFBINcl(A ∪ B) ⊆ n-CyFBINcl(A) ∪ n-CyFBINcl(B)….(1) . Also, n-CyFBIN(A) ⊆ n-CyFBIN(A ∪ B) and n-CyFBIN(B) ⊆ n-CyFBIN(A ∪ B), therefore n-CyFBINcl(A) ∪ n-CyFBINcl(B) ⊆ n-CyFBINcl(A ∪ B)….(2) from (1) and (2) we get n-CyFBIN cl(A ∪ B) = n-CyFBIN cl(A) ∪ n-CyFBINcl(B).

5.N-Cylindrical Fuzzy Bipolar interval Neutrosophic Subspace Topology

In this part, we introduce the definition of n-cylindrical fuzzy bipolar interval neutrosophic subspace topology along with illustrative examples.

Definition 5.1. Let $(\mathrm{U}, \mathrm{t}_{\mathrm{FBIN(U)}})$ be an n-CyFBINTS and $A \subseteq U$. Then the set $\mathrm{t_A} = \{\mathrm{A} \cap \mathrm{U}_i : \mathrm{U}_i \in \mathrm{U}, i \in I\}$ is called n-cylindrical fuzzy bipolar interval neutrosophic subspace topology on A. Thus $(\mathrm{A}, \mathrm{t}_{FBIN(A)})$ is called n-CyFBINT subspace of $(\mathrm{U}, \mathrm{t}_{\mathrm{FBIN(U)}})$.

Example 5.1 Let $\mathrm{t}_{\mathrm{FBN(U)}} = \{0_{FBN}, 1_{FBN}$, A,B$\}$as in Example 4.1 and

$\mathrm{C} =$
$< u_{1,}([0.1,0.3], [0.3,0.5], [0.1,0.3], [-0.3,0], [-0.2, -0.2], [0, -0.1]), \mathrm{u}_2,$
$([0.5,0.7], [0,0.1], [0.2,0.2], [-0.3, -0.3], [-0.2, -0.2], [-0.3, -0.2]) >,$

$\mathrm{C}_1 = C \cap A=$
$< u_{1,}([0.1,0.3], [0.3,0.5], [0.2,0.5], [-0.3,0], [-0.4, -0.4], [0, -0.1]), \mathrm{u}_2,$
$([0.5,0.7], [0,0.15], [0.2,0.3], [-0.3, -0.3], [-0.3, -0.2], [-0.4, -0.2]) >,$

$\mathrm{C}_2 = C \cap B =$
$< u_{1,}([0.1,0.3], [0.3,0.5], [0.3,0.5], [-0.1,0], [-0.4, -0.4], [-0.1, -0.1]), \mathrm{u}_2,$

$([0.5,0.5],[0,0.15],[0.3,0.3],[-0.3,-0.2],[-0.3,-0.2],[-0.5,-0.5]) >$
then $t_{FBN(C)} = \{0_{FBN}, 1_{FBN}, C_1, C_1\}$ n-cylindrical fuzzy bipolar interval neutrosophic topology subspace of $t_{FBN(C)}$

Theorem 5.1 Let $(U, t_{FBIN(A)})$ be an n-CyFBINTS and $A \subseteq U$. Then $t_A$is n-CyFBIN subspace topology on A is an n-CyFBINTS.

Proof. 1) Certainly $0_{FBIN} \in t_{FBIN(U)}$, since $0_{FBIN} \cap A = 0_{FBIN} \in t_{FBIN(A)}$ and $1_{FBIN} \cap A = 1_{FBIN} \in t_{FBIN(A)}$. Thus 0_{FBIN} and $1_{FBIN} \in t_{FBIN(A)}$

2) From **Definition 4.1** we have $G_1, G_2 \in t_{FBIN(U)}, G_1 \cap G_2 \in t_{FBIN(U)}$. Let $A \cap G_1 = A_1$ and $A \cap G_2 = A_2$, where $A_1$and $A_2 \in t_{FBIN(A)}$, then $A_1 \cap A_2$=A$\cap$ $(G_1 \cap G_2) \in t_{FBIN(A)}$.

3) From **Definition 4.1** $G_i \in t_{FBIN(U)}$: $i \in I, \cup G_i \in t_{FBIN(U)}$, let $A_i \in t_{FBIN(A)}$, i$\in$ I, then $\cup A_i = A \cap (\cup G_i) \in t_{FBIN(A)}$. Therefore $t_{FBIN(A)}$ is an n-CyFBINTS.

Conclusion

In this study, we introduce the notion of n-cylindrical fuzzy bipolar interval neutrosophic set. Based on this new concepts, basic theorem and properties are studied. Moreover, an n-cylindrical fuzzy bipolar interval neutrosophic topology interior and closer of an n-cylindrical fuzzy bipolar interval neutrosophic were introduced with some related theorems and examples. Finally, for the purpose of subspace, we used the definition of interval neutrosophic topology space to define an n-cylindrical fuzzy bipolar interval neutrosophic topological subspace, demonstrate some of its properties.

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